

Exercise 1.

Find a normal basis for the splitting field L of $f = T^3 - 11$ over \mathbb{Q} . Derive from this a basis of E/\mathbb{Q} for every subfield E of L .

Exercise 2.

Are the roots of the following polynomials constructible over \mathbb{Q} ?

1. $f_1 = T^4 - 2$
2. $f_2 = T^4 - T$
3. $f_3 = T^4 - 2T$

Exercise 3.

Let K be a subfield of \mathbb{C} and a a root of $T^2 - b \in K[T]$. Show that every element of $K(a)$ is constructible over K . Use this to explain the relationship between the two definitions of constructible numbers from sections 1.1 and 4.7 of the lecture.

Exercise 4.

Construct a regular 5-gon over \mathbb{Q} with ruler and compass.