Exercises for Algebra II Instituto Nacional de Matemática Pura e Aplicada
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To hand in at 31.10.2014 in the exercise class José Ramón Madrid Padilla (monitor)

Exercise 1.
Show that

- the p-adic absolute value $|\cdot|_p : \mathbb{Q} \to \mathbb{R}_{\geq 0}$ where p is a prime number,
- the f-adic absolute value $|\cdot|_f : \mathbb{F}_q(T) \to \mathbb{R}_{\geq 0}$ where $f \in \mathbb{F}_q[T]$ is irreducible, and
- the "infinite absolute value" $|\cdot|_\infty : \mathbb{F}_q(T) \to \mathbb{R}_{\geq 0}$

are indeed absolute values.

Exercise 2.
Prove Lemma 1 from section 5.2 of the lecture.

Exercise 3.
Show that $\{1/n\}_{n\geq 1}$ does not converge w.r.t. any p-adic absolute value. What is the analogues statement for $\mathbb{F}_p(T)$?

Exercise 4.
Let K be a field with a non-archimedean absolute value $|\cdot|$. For a polynomial $f = a_n T^n + \cdots + a_0$ in $K[T]$, define

$$|f| \ = \ \max\{|a_0|, \ldots, |a_n|\}.$$

Show that with this definition, $|\cdot|$ extends uniquely to a non-archimedean absolute value of $K(T)$.
Hint: The property $|fg| = |f| \cdot |g|$ can be shown similarly to Gauss' lemma.