Exercises for Commutative Algebra
List 8 to hand in at 10.5.2017

Exercise 1 (Integral homomorphisms are stable under base change)**.**
Let A be a ring, B an A-algebra and $f : C \to D$ an integral homomorphism of A-algebras. Show that $C \otimes_A B \to D \otimes_A B$ is integral.

Exercise 2.
Let B be an integral domain and A a subring of B. Assume either that $B - A$ is closed under multiplication or that $B = S^{-1}A$ for some multiplicative set S in A. Show that A is integrally closed in B.

Exercise 3 (Jacobson rings)**.**
A ring is *Jacobson* A if every ideal of A is an intersection of maximal ideals. Show the following strenghtening of Theorem 5.5.5 of the lecture: every finitely generated algebra over a field is Jacobson.

Exercise 4.
Let k be a field and $f : A \to B$ a homomorphism between finitely generated k-algebras. Show that the inverse image $f^{-1}(\mathfrak{m})$ of a maximal $\mathfrak{m}$ of B is a maximal ideal of A.

Exercise 5.
Complete the proof of Proposition 1 in section 5.7 of the lecture.

Here are some additional exercises from Atiyah-Macdonald (which are not to hand in): chapter 5, exercises 2, 4, 8 and 14.