A guide to mathematical writing

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The art of writing

Scientific writing is an art, in a similar way as it is to write a novel. There is no precise description of what "good scientific writing" means, and one doesn't have to go far to find contradictory opinions. This goes hand in hand with the saying

"Good writing is a matter of style."

This does, however, *not* mean that good writing *cannot be learned* or that there are *"born writers"*. On the contrary, good writing is a process of learning, and even a skilled writer has to work hard on a text to make it shine.

Even though there is no recipe for good writing, there are

- guidelines that help you to develop your skills and
- rules of what should be and should not be done.

The content

The central aspect of a mathematical text is its content, which should be as accessible to the reader as possible.

The following aspects play a role:

1. the organization of the text;
2. the language;
3. the notation and formulas;
4. the typesetting.

We will examine these aspects one by one in the following.

Organization of a mathematical text

The first task for writing a mathematical text—let it be a research paper, a thesis, a review, or some other format—is to define its purpose. One should have clarity about

- its content (what do I write about?);
- the audience (how do I write the text?);
- the generality and extent (which details do I include?).

Often enough the answers to these questions suggest themselves: for example, one has proven a particular result, which is either interesting to experts only or interesting to a broad mathematical audience.

But sometimes the extent of the content might not be so clear, and then it is wise to reflect about the above questions first.

Approach to writing a mathematical text

Once one has decided the purpose, it might help to proceed along the following steps:

1. work out a draft;
2. write the main body of the text;
3. write an introduction;
4. finalize the text.

1. The draft

The draft organizes the content of the text and its structure, so that one concentrates on the writing in the next step.

For this, the draft should contain

- all statements, including definitions, results, the arguments of their proofs, possibly examples and remarks;
- the organization of the material into sections and environments (such as “Definition” or “Theorem”);
- a coherent notation and terminology for the whole text.

Typically it takes many iterations and rewritings until one has a suitable draft. Note that it usually pays off to work on a solid draft since a lack of planning might create a lot of unnecessary work in the upcoming step.

2. The main body

For the main body, *circular writing* is advisable. This means that one writes the text linearly from A to Z, but revises it regularly. Schematically this could look like:

- write section 1;
- revise section 1; write section 2;
- revise sections 1–2; write section 3;
- revise sections 1–3; write section 4;
- ...

The revision is necessary to achieve a coherence throughout the document. This is the step that profits immensely from a good draft.

3. The introduction

Some aspects for the introduction are the following:

- Begin with a description of the context of the work and its relevance. Mention and cite related work.
- Introduce the main notions of the text and work towards a description of the main results.
- Possibly mention some aspects or techniques of the proofs.
- Possibly outline the organization of the paper, especially in case of a longer text with several results.

Some mathematicians prefer short informative introductions, others enjoy more elaborate introductions. This is largely a matter of taste, but for inexperienced writers the former approach is advisable.

4. Finalizing the text

To finalize the text,

- fix a title,
- write an abstract,
- possibly write a conclusion (depends on the area),
- include acknowledgements (always be generous),
- check the references (and remove those that are not cited),
- insert the authors' (email) addresses,
- possibly insert keywords and MSC codes,
- read once more the complete text,
- and don't forget a final spell check!

Of course, some steps can be taken care of earlier in the writing process.

Non-obstructive use of language

The main principle concerning the language of the writing is that the choice of words and symbols should *not* obstruct the reader.

In particular, one should avoid distractions to the reader through English[1] mistakes or complicated formulations.

As a start, I heartily recommend the reading of two books:

- W. Strunk Jr. and E. B. White: **The Elements of Style**
  A classic text on the art of writing and a must read!
- T. S. Kane: **The New Oxford Guide to Writing**
  A book with all rules for writing in English. There are alternatives, which can be easily found on the web.

[1] We assume here that you write in English, which is the most common language for scientific writing. Of course, "English" can be replaced by any other language.

Some rules

This implies some simple rules:

- Avoid mistakes in English: check for typos (i.e. run a spell check), check the punctuation, proof read the text with an eye on the grammar.
- Use short sentences and simple formulations, unless you are very safe and experienced in English.
- It is OK to mix personal[2] ("we consider a group G") and impersonal formulations ("let G be a group" or "one finds that...") if this allows for a better text flow.

[2] Note that one uses "we" in the sense of "the reader and I", i.e. "we" is also used in the case of a single author.

Absence of time flow

There is no time flow in a scientific text, but scientific findings are considered as time independent truth.

In so far, a mathematical text should be writing entirely in present tense, and time modifiers such as “now” and “afterwards” should be strictly avoided.

For example:

bad	good
Now we consider...	We continue to...
Let $x > 0$... Then let $x < 0$...	Let $x > 0$... Next let $x < 0$...
The author showed in [2]...	The author shows in [2]...
In section 2, we will prove...	In section 2, we prove...

Different English conventions

Different conventions in English are used and acceptable.

For example:

- British spelling versus American spelling;
- comma rules: e.g. when to put a comma after a introductory clause, usage of a comma before "where", the Oxford comma, et cetera;
- frequent (and thus accepted) violation of proper English: e.g. "associated to" instead of "associated with"; omittance of a comma after "i.e." and "e.g."; "let a (resp. b) be an element of A (resp. B)" instead of "let a and b be elements of A and B, respectively."

Whatever convention you follow, make sure you follow it consistently throughout the text!

Some more tips

Here is some further advice:

- Avoid phrases such as "It is trivial / clear / obvious / easy". This might insult the reader for whom this is not obvious. Either simply omit such a phrase (in case it is really obvious; ask yourself!), or give a short argument.
- There are very few abbreviations that are OK for scientific writing. You will almost never face other instances than "i.e." and "e.g.".[3] In particular, the abbreviations "iff.", "s.t.", "w.r.t." and "wlog." are *not OK*.
- Never start a sentence with a symbol or a formula.[4]
- Don't use footnotes with formulas, e.g. avoid "$\sqrt{y} = x$[5]".

[3] Confer the previous slide for "resp.", which forms an exception.

[4] The main reason is that a full stop followed by a capitalized word helps the reader to find the end of a sentence.

[5] In this example the footnote label looks like an exponent.

Mathematical content

In general, the mathematical content should be clearly structured and presented. The way of writing depends on the area of mathematics and can vary in several aspects:

- the amount of symbols, formulas and diagrams;
- the formality of statements and arguments;
- the amount of details in proofs;
- the subdivision of proofs into auxiliary results.

Generally speaking, it is advisable for the novice writer to

- write in a formal and mathematically precise way;
- give detailed proofs;
- use a clear logical structure and rather more than less auxiliary statements.

Mathematical accuracy

An important aspect of the text is its mathematical accuracy.

This involves the following:

- Introduce carefully all notation in the correct order (don't use a notion before it is introduced).
- Explain all symbols and state all hypotheses in a theorem.
- If you have a standing notation or hypothesis, then explain this at a visible position (e.g. at the beginning of a section) and repeat it in key positions (e.g. in the main theorem).
- Don't use the same symbol for different things.

Proofs

If the eloquence of writing is the paper's face, then the thoroughness of proofs is its heart.

Some guidelines for writing proofs are the following:

- Before writing up a proof, make sure that you understand every step, i.e. include a detailed sketch in your draft.
- By default, you should explain all details in a proof. There are exceptions, e.g. for an overview paper, as is the case in this course.
- Give a precise reference to every statement that you use: either include the label of the statement (e.g. "by Lemma 3.4 in [2]") or a precise position (e.g. "as explained in the second paragraph of page 8 in [3]").
- Mention explicitly where you use the hypotheses of your claim in the proof.

LaTeX

To dissolve any doubt: there is no alternative to LaTeX for writing a mathematical text, and the same holds for many other scientific areas such as computer science and physics.

There a numerous good introductions to LaTeX on the internet; for instance, search for "`introduction to latex`."

For research papers, the most common article class is `amsart`. Its convenient layout can be used 'out of the box' with the line[6]

```
\documentclass[a4paper,12pt]{amsart}
```

For example files, search for "`latex amsart templates`."

[6] The option 'a4paper' chooses the synonymous paper format, the option '12pt' selects the font size. Note that the standard font size is 10pt, but 12pt is gentle to collegues that prefer to read a larger font and to those that like to print two pages on one sheet.

Useful packages

The following is a list of useful latex packages:

- `geometry` (define the margins);
- `verbatim` (comment out whole paragraphs);
- `microtype` (better spacing at the end of lines);
- `fontenc`, `inputenc` and `babel` (hyphenation and more);
- `hyperref` (clickable links in pdf);
- `xcolor` and `graphicx` (colours and graphic commands);
- `tikz` and `tikz-cd`; alternatively `xymatrix` (for diagrams);
- `todonotes` (write remarks into margins);
- `mathptmx` (times font);

These packages are included with commands like

```
\usepackage[top=3cm]{geometry}
```

The references are easiest managed with Bibtex or Biblatex.

Useful commands

With `\DeclareMathOperator` one can define commands for math mode that display some upright words with a good spacing around the words. For example, "Hom" in

$$\text{"every } f \in \operatorname{Hom}(G,H) \text{ satisfies } f(g+h) = f(g) + f(h)\text{."}$$

With `\newcommand` one can define shortcuts for longer commands, such as `\newcommand\C{\mathbb{C}}` to display $\mathbb{C}$ with the command `\C`.

With the package `amsthm` (included in the amsart class), one can define theorem-like environment with the command `\newtheorem`. Note that it helps the reader to find statements if all environments share a common counter, such as Definition 1.1—Lemma 1.2—Example 1.3—Theorem 1.4, and so forth.

Formulas

There are two types of math modes: 'text style' and 'display style'; e.g. `$y=2x$` creates "$y = 2x$" and `\[y=2x\]` creates

$$y = 2x.$$

Aim for a balanced use of inline and displayed formulas. Note that one can give a displayed formulas a citable equation number; cf. the commands `\tag`, `\label` and `\eqref`.

Balance the spaces and font sizes in displayed formulas; e.g. compare

$$y = \frac{1}{2a}(-b + \sqrt{b^2 - 4ac}) \text{ or } y = \frac{1}{2a}(-b - \sqrt{b^2 - 4ac})$$

with

$$y = \frac{1}{2a}\Big(-b + \sqrt{b^2 - 4ac}\Big) \quad \text{or} \quad y = \frac{1}{2a}\Big(-b - \sqrt{b^2 - 4ac}\Big).$$

Further reading

Please note that these slides express my personal view on good writing. One can easily find contradictory statements.

Some further reading that is openly available on the web:

- Paul R. Halmos: **How to write mathematics**
- Steven G. Krantz: **How to write your first paper**
- Kevin P. Lee: **A guide to writing mathematics**
- Bjorn Poonen: **Practical suggestions for mathematical writing**

The very amusing, but insightful, talk **How to write mathematics badly** by Jean Pierre Serre can be found at `https://www.youtube.com/watch?v=ECQyFzzBHlo&feature=youtu.be`.