Exercises for Algebra II
Series 1
To hand in at 10.8.2015 in class

Instituto Nacional de Matemática Pura e Aplicada
Carolina Araujo and Oliver Lorscheid
Roberto Alvarenga Jr. (monitor)

Exercise 1.
Let r be a positive real number. Show that $h = \sqrt{r}$ is contructible.
Hint: You are allowed to use classical geometric theorems like the theorem of Thales or the theorem of Pythagoras.

Exercise 2.
Construct the following regular n-gons with ruler and compass:

1. a regular 2^r-gon for $r \geq 2$;
2. a regular 3-gon;
3. a regular 5-gon.

Exercise 3.
Prove Cardano's formula: given an equation $x^3 + px + q = 0$ with real coefficients p and q such that $\Delta = q^2/4 + p^3/27 > 0$, then

$$x \quad = \quad \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}} \; + \; \sqrt[3]{-\frac{q}{2} - \sqrt{\Delta}}$$

is a solution.

Exercise 4.
Find all solutions for $x^4 - 2x^3 - 2x - 1 = 0$.
Hint: Use Ferrari's formula.

Exercise 5 (very difficult; not to hand in)**.** Find solutions to the following classical problems:

1. Given a positive real number r, is it possible to construct the cube root $\sqrt[3]{r}$?
2. Given an angle φ, is it possible to construct $\varphi/3$?
3. Given a circle with area A, is it possible to construct a square with area A?