**Exercises for Algebra 2**
**List 1** To hand in at 24.8.2020

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**Exercise 1.**
Let $P$ be a point in $\mathbb{R}^2$ with coordinates $x$ and $y$. Show that $P$ is constructible from a given set of points $0, 1, P_1, \ldots, P_n$ if and only if $x$ and $y$ are constructible (considered as points $(x, 0)$ and $(y, 0)$ of the first coordinate axis in $\mathbb{R}^2$). Conclude that the point $P_1 + P_2$ (using vector addition) is constructible from $0, 1, P_1, P_2$.

**Exercise 2.**
Let $r$ be a positive real number. Show that $h = \sqrt{r}$ is constructible from 0, 1 and $r$.
*Hint:* You are allowed to use classical geometric theorems like the theorem of Thales or the theorem of Pythagoras.

**Exercise 3.**
Construct the following regular $n$-gons with ruler and compass:

1. a regular $2^r$-gon for $r \geq 2$;
2. a regular 3-gon;
3. a regular 5-gon.

**Exercise 4.**
Prove Cardano's formula: given an equation $x^3 + px + q = 0$ with real coefficients $p$ and $q$ such that $\Delta = q^2/4 + p^3/27 > 0$, then

$$x = \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\Delta}}$$

is a solution.

**Exercise 5.**
Find all solutions for $x^4 - 2x^3 - 2x - 1 = 0$.
*Hint:* Use Ferrari's formula.

**Exercise 6** (very difficult; not to hand in)**.** Find solutions to the following classical problems:

1. Given a positive real number $r$, is it possible to construct the cube root $\sqrt[3]{r}$?
2. Given an angle $\varphi$, is it possible to construct $\varphi/3$?
3. Given a circle with area $A$, is it possible to construct a square with area $A$?